

Superfluidity and entrainment in neutron-star crusts

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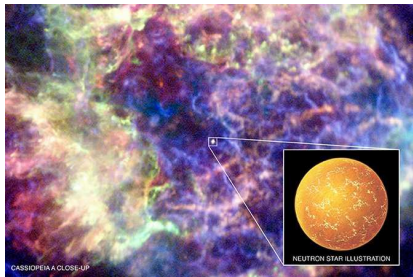
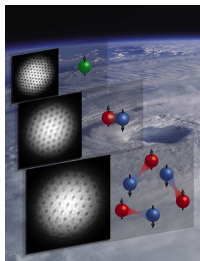
Beijing, August 2012

What is superfluidity and why is it relevant for neutron stars?

Superfluidity is a macroscopic quantum phenomenon.

A superfluid has no viscosity and its angular momentum is quantized into vortex lines.

Vortices in cold gases (MIT)



Cassiopeia A (NASA)

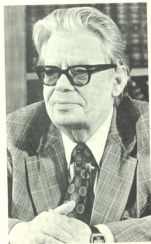
Superfluidity affects the evolution of neutron stars:
cooling, oscillations,
precession, pulsar glitches, etc

Nuclear superfluidity in neutron stars

The BCS theory was applied to nuclei by Bohr, Mottelson, Pines and Belyaev

Phys. Rev. 110, 936 (1958).

Mat.-Fys. Medd. K. Dan. Vid. Selsk. 31 , 1 (1959).



N.N. Bogoliubov, who developed a microscopic theory of superfluidity and superconductivity, was the first to explore its application to nuclear matter.

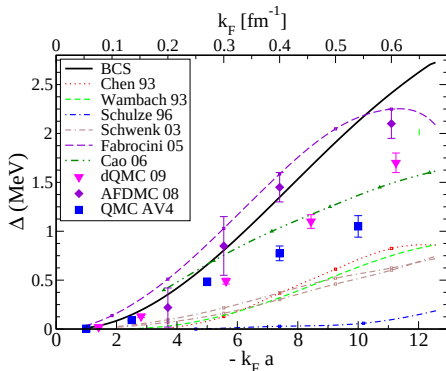
Dokl. Ak. nauk SSSR 119, 52 (1958).

Superfluidity in neutron stars was suggested long ago (before the actual discovery of neutron stars) by Migdal in 1959. It was first studied by Ginzburg and Kirzhnits in 1964.
Ginzburg and Kirzhnits, Zh. Eksp. Teor. Fiz. 47, 2006, (1964).

Superfluidity in neutron matter

Most microscopic studies of nuclear superfluidity have focused on uniform neutron matter.

At $T < T_c$, neutrons arrange themselves in Cooper pairs like electrons in BCS superconductors.

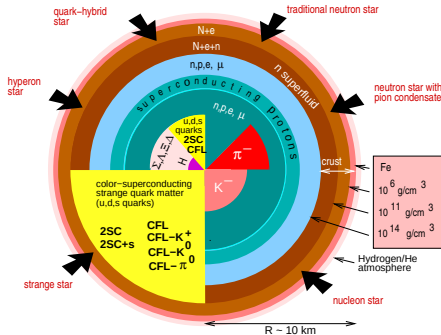


Recent calculations using different methods tend to predict similar results, at least at not too high densities.

*Gezerlis & Carlson,
Phys.Rev.C81,025803(2010).*

Superfluidity and superconductivity in neutron stars

But neutron stars are not only made of neutrons! As a consequence, they could contain various kinds of superfluids and superconductors.

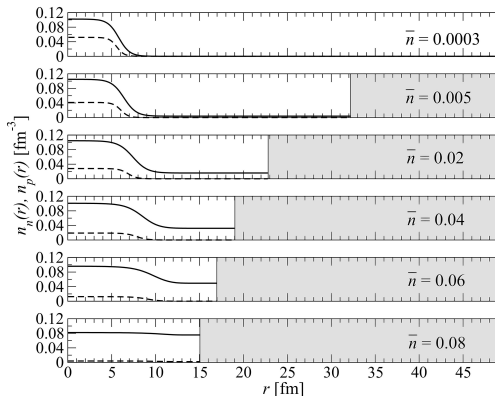


picture from
F. Weber

All microscopic models of neutron stars predict neutron superfluidity in their inner crust.

Structure of the inner crust of neutron stars

However the inner crust of a neutron star is neither made of pure neutron matter nor homogeneous!

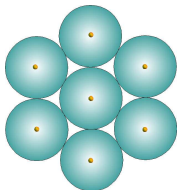


Extended Thomas-Fermi calculations including proton quantum shell effects via the Strutinsky integral for the generalized Skyrme functional BSk21.

See poster 298.

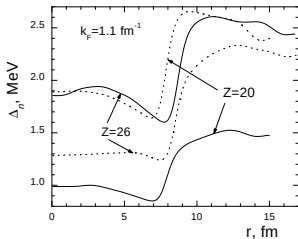
Pearson, Chamel, Goriely, Ducoin, Phys. Rev. C 85, 065803 (2012).

Superfluidity in neutron-star crusts (I)



Most studies are based on the **Wigner-Seitz approximation**: the crust is decomposed into independent cells, each of which is viewed as a big nucleus.

However, this approach is limited by **spurious shell effects**.
Chamel et al., Phys.Rev.C75(2007)055806.



Shell effects $\propto 1/R^2$ are very large at the crust bottom and are enhanced by the self-consistency.

Baldo et al., Eur.Phys.J. A 32, 97(2007).

Nuclear band theory

Pairing is a non-local phenomenon which therefore requires a consistent treatment of nuclear clusters and unbound neutrons.

"I found to my delight that the wave differed from the plane wave of free electrons only by a periodic modulation." F. Bloch



Band theory of solids in a nut shell:

$$\varphi_{\alpha\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\alpha\mathbf{k}}(\mathbf{r})$$

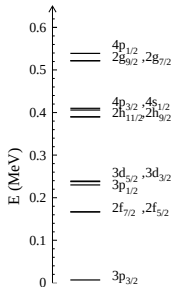
$$u_{\alpha\mathbf{k}}(\mathbf{r} + \mathbf{T}) = u_{\alpha\mathbf{k}}(\mathbf{r})$$

- $\alpha \rightarrow$ rotational symmetry around lattice sites ($\rightarrow$ clusters)
- $\mathbf{k} \rightarrow$ translational symmetry ($\rightarrow$ unbound neutrons)

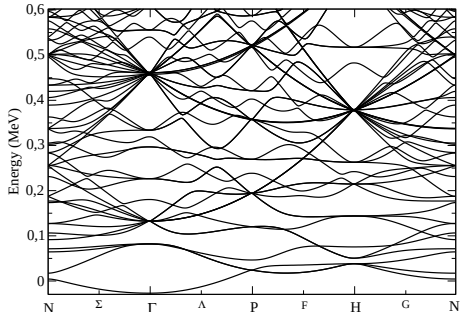
Example of neutron band structure

Body-centered cubic crystal of zirconium like clusters with $N = 160$ (70 unbound) and $\bar{\rho} = 7 \times 10^{11} \text{ g.cm}^{-3}$

W-S approximation



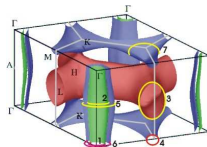
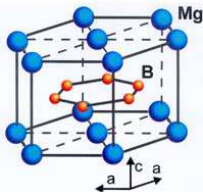
band theory



Chamel et al, *Phys.Rev.C*75 (2007), 055806

Analogy with terrestrial multi-band superconductors

Multi-band superconductors were first studied in 1959 but clear evidence were only found in 2001 with the discovery of MgB_2 (two-band superconductor)

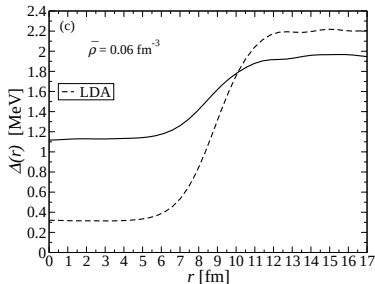
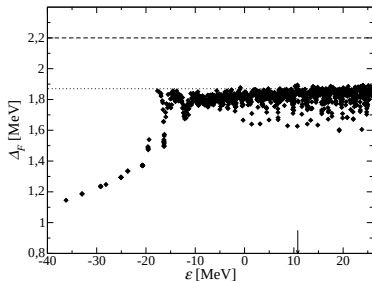


In neutron-star crusts,

- the number of bands can be huge $\sim$ up to a thousand!
- both intra- and inter-band couplings must be taken into account

Superfluidity in neutron-star crusts (II)

Pairing gaps and pairing fields at $\bar{n} = 0.06 \text{ fm}^{-3}$ for BSk16.



Chamel et al., *Phys.Rev.C*81,045804(2010).

- The presence of inhomogeneities lowers T_c by $\sim 20\%$.
- Pairing correlations are highly non-local (strong proximity effects).

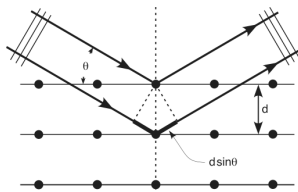
Neutron diffraction and entrainment

Despite the absence of viscous drag, the crust can still resist the flow of the superfluid due to non-local and non-dissipative entrainment effects.

Chamel, Phys. Rev. C85, 035801 (2012).

A neutrons with wavevector $\mathbf{k}$ can be elastically scattered by the lattice (Bragg diffraction) if

$$2d \sin \theta = N\lambda$$



- Neutrons that are reflected do not propagate: $\mathbf{v} = 0$
- Others do propagate but with $\mathbf{v} = \hbar \mathbf{k} / m_n^*(k)$

How “free” are neutrons in neutron-star crusts?

On average most neutrons are actually entrained by the crust
Chamel, Phys. Rev. C 85, 035801 (2012).

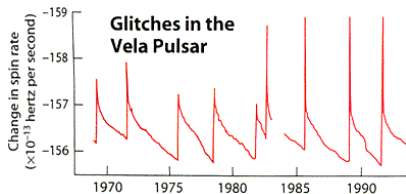
$\bar{n}$ (fm ⁻³)	n_n^f/n_n (%)	n_n^c/n_n^f (%)
0.0003	20.0	82.6
0.001	68.6	27.3
0.005	86.4	17.5
0.01	88.9	15.5
0.02	90.3	7.37
0.03	91.4	7.33
0.04	88.8	10.6
0.05	91.4	30.0
0.06	91.5	45.9
0.08	104	64.8

The density $n_n^c = n_n^f/m_n^*$ of “conduction” neutrons can be much smaller than the density n_n^f of “free” neutrons!

Entrainment impacts our understanding of pulsar glitches

Pulsar glitches and superfluidity

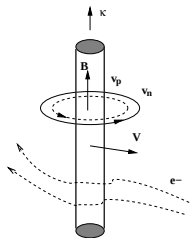
Sometimes pulsars may suddenly spin up. These glitches are followed by a relaxation over days to years thus providing strong evidence of neutron-star superfluidity.



Superfluidity is also expected to play a key role in the mechanism of large glitches (e.g. catastrophic unpinning of superfluid vortices)

Entrainment and dissipation in neutron star cores

Historically the **long post-glitch relaxation** was the first evidence of neutron-star superfluidity. But...



Due to (non-dissipative) mutual entrainment effects, neutron vortices carry a **fractional magnetic quantum flux**

Sedrakyan and Shakhbasyan, Astrofizika 8 (1972), 557; Astrofizika 16 (1980), 727.

picture from K. Glampedakis

The core superfluid is strongly coupled to the crust due to electrons scattering off the magnetic field of the vortex lines.

Alpar, Langer, Sauls, ApJ282 (1984) 533

Andersson, Sidery, Comer, MNRAS368 (2006), 162

Pulsar glitches and entrainment

Glitches are usually interpreted as **sudden tranfers of angular momentum between the superfluid in the crust and the rest of star**. However this superfluid is also entrained !

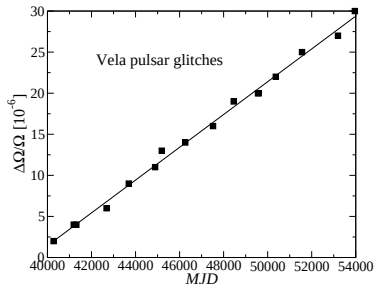
$$\begin{aligned} J_s &= I_{ss}\Omega_f + I_{sc}\Omega_c \\ J_c &= I_{cc}\Omega_c + I_{sc}\Omega_f \Rightarrow \frac{(I_s)^2}{I_{ss}I} \geq A_g \frac{\Omega}{|\dot{\Omega}|}, \end{aligned} \quad A_g = \frac{1}{t} \sum_i \frac{\Delta\Omega_i}{\Omega}$$

Chamel&Carter,MNRAS368,796(2006)

Application to the Vela pulsar:

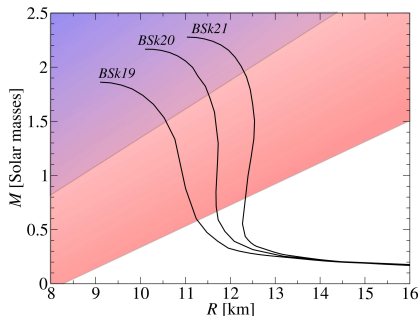
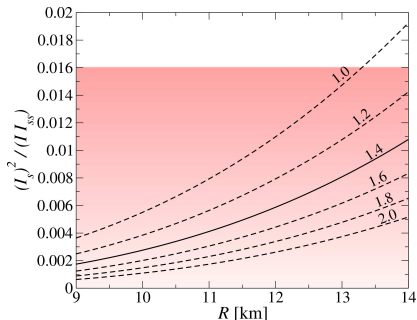
$$A_g \simeq 2.25 \times 10^{-14} \text{ s}^{-1}$$

$$\frac{(I_s)^2}{I_{ss}I} \geq 1.6\%$$



Pulsar glitch constraint

Calculated ratio $(I_s)^2/I_{ss}I$ and corresponding constraint on pulsar structure:



Inferred mass of Vela for the stiffest EoS: $M \lesssim 0.8M_{\odot}$.

Due to entrainment, the superfluid in the crust does not carry enough angular momentum to explain large glitches.

Chamel, submitted

Summary and perspectives

- Superfluidity in neutron-star crusts is a highly non-local phenomenon
- Despite the absence of viscous drag, the superfluid can still be entrained by the crust due to Bragg scattering
- Strong entrainment may have implications for various astrophysical phenomena (e.g. pulsars glitches, QPOs in SGRs, thermal X-ray emission, etc)

Open issues:

- effects of collective excitations on pairing and vice versa ?
- impact of disorder (quantum and thermal fluctuations, impurities, defects, etc) on entrainment ?