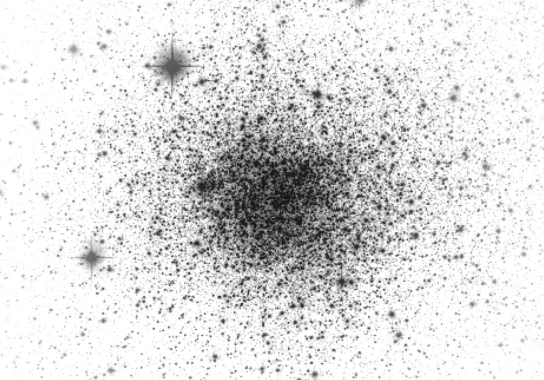
Constraining the luminosity function parameters and population size of radio pulsars in globular clusters



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Introduction

- Globular clusters host low-mass X-ray binaries (LMXBs)
- LMXBs $\Rightarrow$ millisecond pulsars (MSPs)
- What can a globular cluster pulsar sample tell us about luminosity function parameters, population size?

- Radio luminosity distribution – lognormal (Faucher-Giguère & Kaspi 2006, Bagchi *et al.* 2011):

$$f(\log L) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\log L - \mu)^2}{2\sigma^2}}$$

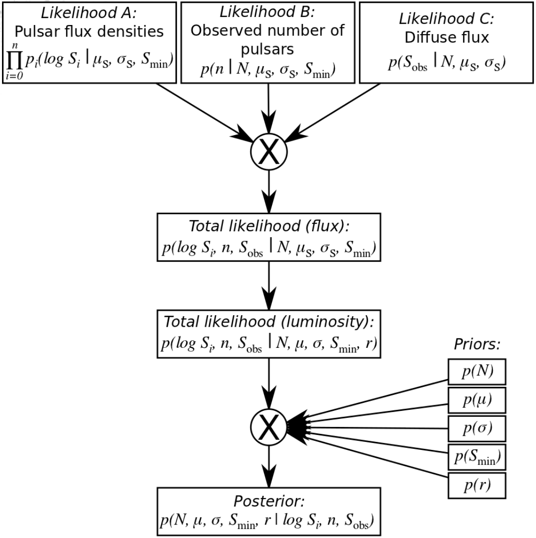
- Pseudoluminosity: $L = S\ r^2$
- Flux density distribution

$$f(\log S) = \frac{1}{\sigma_S\sqrt{2\pi}} e^{-\frac{(\log S - \mu_S)^2}{2\sigma_S^2}}$$

Bayesian Parameter Estimation

$$p(\boldsymbol{\theta}|D, M) \propto p(D|\boldsymbol{\theta}, M)p(\boldsymbol{\theta}|M)$$

$\boldsymbol{\theta} \rightarrow$ Parameters (μ, σ, N, ...)
$D \rightarrow$ Data (n, $\{S_i\}$, $S_{\rm obs}$)
$M \rightarrow$ Model (Form of luminosity function – lognormal)


Likelihood A:
Pulsar flux densities
$\prod_{i=0}^{n} p_i(\log S_i \mid \mu_S, \sigma_S, S_{min})$
Likelihood B:
Observed number of pulsars
$p(n \mid N, \mu_S, \sigma_S, S_{min})$
Likelihood C:
Diffuse flux
$p(S_{obs} \mid N, \mu_S, \sigma_S)$
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Total likelihood (flux):
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Priors:
$p(N)$
$p(\mu)$
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Posterior:
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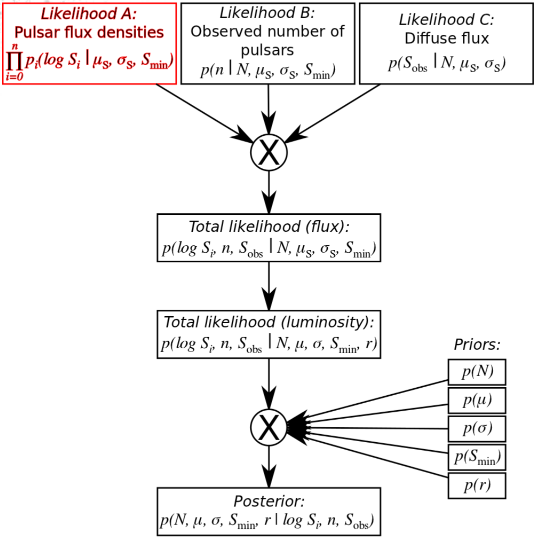
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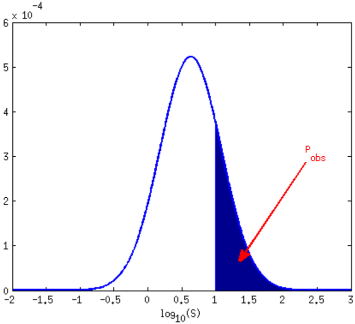
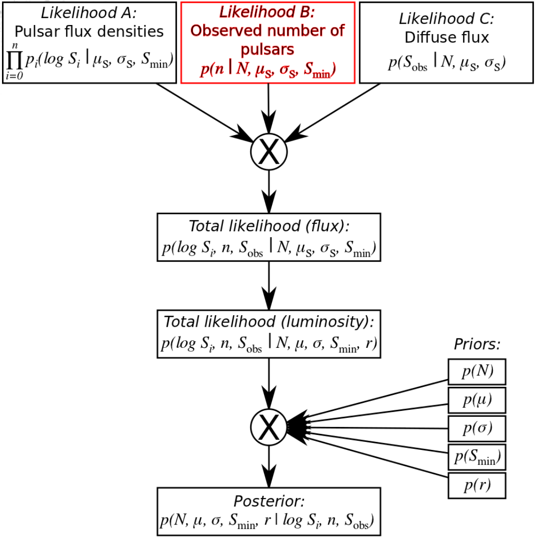
Likelihood A: Pulsar flux densities

$$\prod_{i=1}^{n} p_i(\log S_i | \mu_S, \sigma_S, S_{\min}) = \prod_{i=1}^{n} \frac{1}{p_{\rm obs}\sigma_S\sqrt{2\pi}} e^{-\frac{(\log S_i - \mu_S)^2}{2\sigma_S^2}}$$



$\log S_{\rm min}$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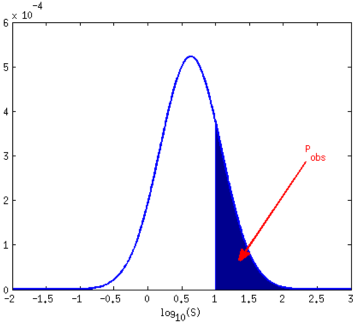
Likelihood B: Number of observed pulsars

$$p(n|N,\mu_S,\sigma_S,S_{\min}) = \frac{N!}{n!(N-n)!}\, p_{\rm obs}^{n}\, (1-p_{\rm obs})^{N-n}$$



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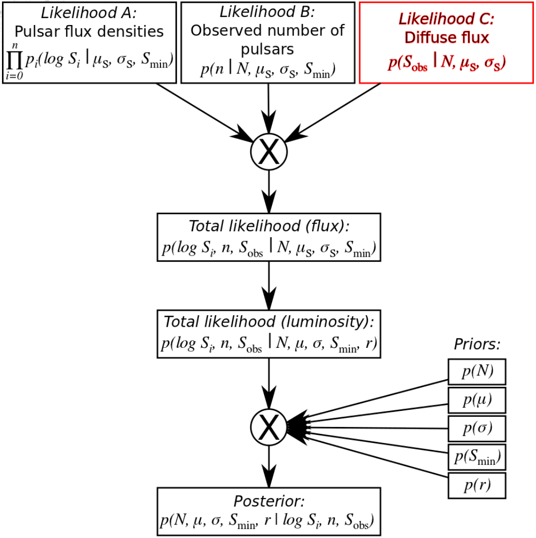
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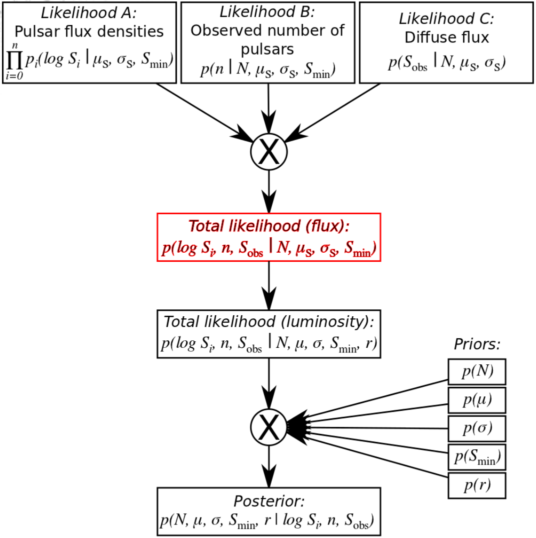
Likelihood C: Diffuse emission

$$p(S_{\rm obs}|N, \mu_S, \sigma_S) = \frac{1}{\sigma_{\rm diff}\sqrt{2\pi}}\; e^{-\frac{(S_{\rm obs}-S_{\rm diff})^2}{2\sigma_{\rm diff}^2}}$$

$$S_{\rm diff} = N\langle S\rangle$$

$$\sigma_{\rm diff} = \sqrt{N}\,\mathrm{SD}(S)$$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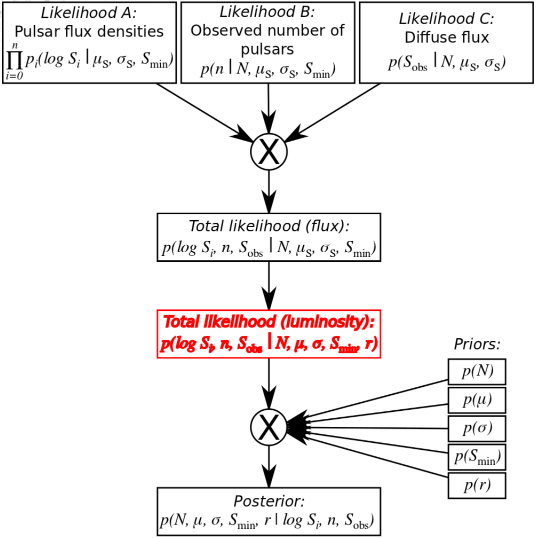
Total likelihood (flux domain)

$$p(\log S_i, n, S_{\mathrm{obs}} | N, \mu_S, \sigma_S, S_{\mathrm{min}}) = \left[\prod_{i=1}^{n} p_i(\log S_i | \mu_S, \sigma_S, S_{\mathrm{min}}) \right] \times p(n | N, \mu_S, \sigma_S, S_{\mathrm{min}}) \times p(S_{\mathrm{obs}} | N, \mu_S, \sigma_S)$$



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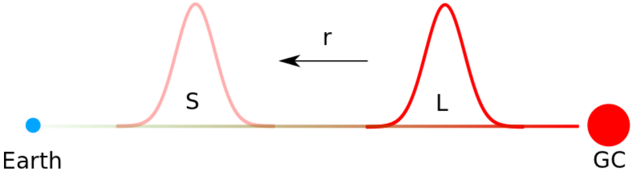
Transformation to luminosity domain



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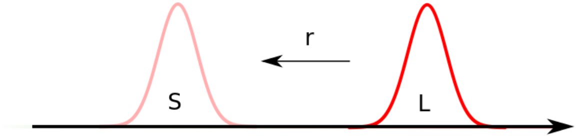
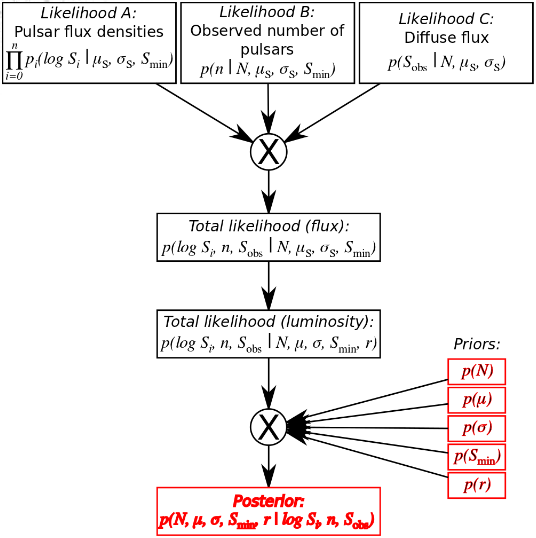


$$\log L = \log S + 2 \log r$$

$$\mu = \mu_S + 2 \log r$$

$$\sigma = \sigma_S$$

$$p(\log S_i, n, S_{\rm obs} | N, \mu, \sigma, S_{\rm min}, r) = p(\log S_i, n, S_{\rm obs} | N, \mu_S, \sigma_S, S_{\rm min})$$



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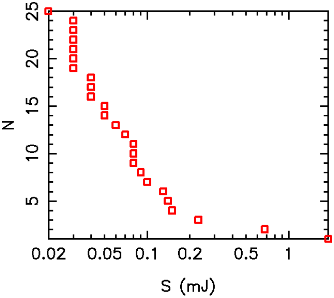
$$p(N, \mu, \sigma, S_{\rm min}, r | \log S_i, n, S_{\rm obs}) \propto \; p(\log S_i, n, S_{\rm obs} | N, \mu, \sigma, S_{\rm min}, r)$$
$$\times\, p(N)\; p(\mu)\; p(\sigma)\; p(S_{\rm min})\; p(r)$$

$$p(\theta_1 | D) = \int p(\boldsymbol{\theta} | D)\; d\boldsymbol{\theta}_{\mathbf{2-M}}$$

Data and Priors for Terzan 5

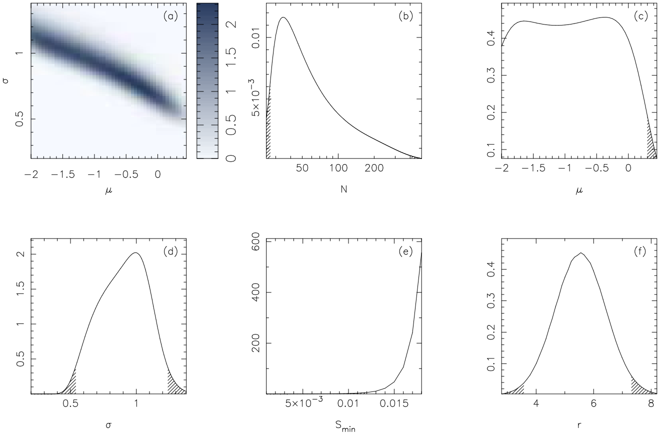
Data	
$\{S_i\}$	$\rightarrow$
n	25
$S_{\rm obs}$	5.2 mJy
	(Fruchter & Goss 2000)
Priors	
μ	Uniform in $[-2.0, 0.5]$
σ	Uniform in $[0.2, 1.4]$
$S_{\rm min}$	Uniform in $(0, 0.02]$
N	Uniform in $[25, 500]$
d	Gaussian, with mean 5.5 kpc and SD 0.9 kpc
	(Ortolani *et al.* 2007)



Ransom *et al.* 2005,
Hessels *et al.* 2006

Results

Terzan 5, wide priors

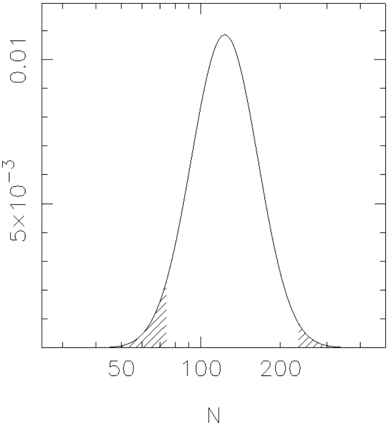


Terzan 5, narrow priors

μ: Uniform in $[-1.19, -1.04]$

σ: Uniform in $[0.91, 0.98]$

(Boyles *et al.* 2011)



$$N = 133^{+101}_{-58}$$

Summary and Conclusions

- A new Bayesian technique to constrain luminosity function parameters and pulsar population size
- Prior information helps obtain better constraints on N
- More data (more individual pulsar and diffuse flux measurements) will help further parameter estimation, cluster comparison

Thank You

Paper: `arxiv.org/abs/1207.5732`

Source Code: `psrpop.phys.wvu.edu/gcbayes`