Relativistic Anisotropic Strange Stars in Pseudo Spheroidal Space Time

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Introduction and Motivation

The analysis of very compact astrophysical objects has been a key issue in relativistic astrophysics for the last few decades.

The estimated masses and radii of many compact objects such as X-ray pulsar Her X-1, X-ray burster 4U 1820-30, millisecond pulsar SAX J 1808.4-3658, X-ray sources 4U 1728-34, PSR 0943+10 and RX J185635-3754 are not compatible with the standard neutron star models.

A strange quark matter may be useful to understand the observed physical features of some of these compact objects [1,2,3,4,5].

The matter densities of these compact objects are normally above the nuclear matter density. It has both maximum mass and radius less than those for neutron stars, with higher compactification factor (ratio of mass to radius).

It is physically realistic to consider an ultra-compact star with two different pressures inside [6], namely, the radial pressure and the tangential pressure incorporating the anisotropy.

It is considered [7,8,9] that compact stars may be made up of quark matter which may be formed in two different ways :

(i) quark-hadron phase transition in the early universe, and
(ii) conversion of neutron star into strange star ones at ultrahigh densities.

The theories of strong interaction with quark bag model, strange quark matter may be useful in order to obtain a relevant equation of state (EOS).

Here we assume that the quarks are mass less and non-interacting giving the quark pressure

$$p_q = \frac{1}{3}\rho_q$$

where ρ_q is the quark energy density.

The total energy density $\rho = \rho_q + B$
and the total pressure $p = p_q - B$ where B is Bag constant.

The equation of state (EOS) for the strange quark matter [10] is given by:

$$p = \frac{1}{3}(\rho - 4B) \text{------}(1)$$

In the MIT Bag model [11] and in the original version of fuzzy Bag model the non-perterbative QCD vacuum is parameterized by a constant B in the Lagrangian density.

For these stars quark confinement is important which is described by the energy term proportional to the volume[12] .

In this model, the constituent quark matter is considered to be mass less u, d quarks and massive quarks and electrons. The quarks are considered to be degenerate Fermi gases, which may exist only in a region of space with a vacuum energy density B (called the Bag constant).

In this work, we have investigated the role of pressure anisotropy of a compact star relating it with the value of Bag parameter in the framework of Vaidy-Tikekar model for super dense star [13,14].

In this approach the model of a super dense star is obtained by stipulating a law for variation of density of its matter content which follows from prescribing a geometry characterized by two curvature parameters for the physical space of the configuration which is a departure from the spherical geometry of a uniform density configuration.

The equation of state of matter content follows from the system of associated Einstein field equations which in a number of specific cases is found to approximate to linear EOS connecting pressure and density [12].

Einstein Fields Equation

The space time in the interior of a spherically symmetric,cold compact star in equilibrium is described by

$$ds^2 = -e^{2\mu(r)}dt^2 + e^{2\nu(r)}dr^2 + r^2[d\theta^2 + \sin^2\theta d\phi^2] \text{--}(2)$$

Where $\mu(r)$ and $\nu(r)$ are the two unknown metric functions.

The interior matter content of the star is prescribed to be that of a fluid with anisotropic pressures with the energy momentum tensor $T_{ij} = (-\rho, p_r, p_\perp, p_\perp) \text{------}(3)$

where ρ is the energy-density, p_r is the radial pressure, $p_\perp$ is the tangential pressure and $\Delta = p_\perp - p_r$ is the measure of pressure anisotropy [13,15,16,17,18] in this model, which depends on metric potential $\mu(r)$ and $\nu(r)$
The Einstein field equation

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi G T_{ij} \text{--------------}(4)$$

where R_{ij} is Ricci tensor and R is the Ricci scalar. The Einstein field equation relates the metric parameters $\mu(r)$ and $\nu(r)$ of the space time with the dynamical variables of its physical content which reduces to the following system of three equations :

$$\rho = \frac{(1-e^{-2\mu})}{r^2} + \frac{2\mu' e^{-2\mu}}{r} \text{------------}(5)$$

$$p_r = \frac{2\nu' e^{-2\mu}}{r} - \frac{(1-e^{-2\mu})}{r^2} \text{------------}(6)$$

$$p_\perp = e^{-2\mu}[\nu'' + \nu'^2 - \nu'\mu' + \frac{\nu'}{r} - \frac{\mu'}{r}] \text{-------}(7)$$

Where we have used $8\pi G = 1$

The geometry of a more realistic star with variable matter density is expected to be departure from 3-spherical geometry. The Vaidya - Tikekar models are obtained by prescribing[14,19]

$$e^{-2\mu} = \frac{1 + \frac{ar^2}{R^2}}{1 + \frac{r^2}{R^2}} \text{----------}(8)$$

where 'a' and 'R' are two different parameters, 'a' being the spheroidicity parameter and 'R' is expressed in Km. The geometry of the physical 3-space of the star is that of a 3-Pseudo spheroid and the parameter 'a' is related with the eccentricity of the 3-Pseudo spheroid.

In view of the field equation (5) this is equivalent to prescribing the law for variation of matter density at the centre of the star for the physical content of the star

The variation of ρ is governed by two parameters 'a' and 'R'. The matter density has maximum value at the center from which it decreases radialy outward.

$$\rho_0 = \frac{3(a-1)}{R^2} \text{------}(9)$$

Equations (6) and (7) determine the pressures along radial and transverse directions in terms of $\nu(r)$ and these parameters at all points of the star.

If the nature of anisotropy is known these equations determine the metric variable $\nu(r)$.

Now using equations (6) and (7), ones obtains a second order differential equation in 'x'

$$(1-a+ax^2)\Psi_{xx} - ax\Psi_x + a(a-1)\Psi - \frac{\Delta R^2(1-a+ax^2)^2}{(x^2-1)}\Psi = 0 \qquad \text{------}(10)$$

Where $\Psi = e^{\nu(r)}$ with $x^2 = 1 + \frac{r^2}{R^2}$

For simplicity we choose the anisotropic parameter 'Delta' as follows, $\Delta = \frac{\alpha a^2(x^2-1)}{R^2(1-a+ax^2)^2}$

The above relation is chosen so that the regularity at the center is ensured and to obtain relativistic solution similar to that obtained by R.Tikekar et al [20] for the field equations (5)-(7) Using the transformation, $z = \sqrt{\frac{a}{a-1}}x$ eq. (10) can be written as

Where $\beta^2 = 2 - \lambda + \alpha\lambda$ is a constant $(z^2-1)\Psi_{zz} - z\Psi_z - (\beta^2-1)\Psi = 0 \text{------}(11)$

General Solution

The general solution of eq. (11) [20] is given below in two cases

Case-I : In this case the value of λ and α are such that $\beta = \sqrt{(2-\lambda+\alpha\lambda)}$ is positive and the solution is

$$\Psi = C\left[\beta\sqrt{(z^2-1)}\cosh(\beta\eta) - z\sinh(\beta\eta)\right] + D\left[\beta\sqrt{(z^2-1)}\sinh(\beta\eta) - z\cosh(\beta\eta)\right] \text{......}12(a)$$

Case-II : In this case the value of λ and α are such that $\beta = \sqrt{(\lambda-2-\alpha\lambda)}$ is positive and the solution is

$$\Psi = C\left[\beta\sqrt{(z^2-1)}\cos(\beta\eta) - z\sin(\beta\eta)\right] + D\left[\beta\sqrt{(z^2-1)}\sin(\beta\eta) - z\cos(\beta\eta)\right] \text{.....}\ 12(b)$$

Where C and D are two constants to be determined from boundary condition and $z = \cosh(\eta)$

Physical Parameters

The physical parameters of a general relativistic star are given by

$$\rho = \frac{1}{R^2(z^2-1)}\left[1 + \frac{2}{(\lambda-1)(z^2-1)}\right] \text{---------------}(13)$$

$$p_r = -\frac{1}{R^2(z^2-1)}\left[1 - \frac{2z}{(a-1)}\frac{\Psi_z}{\Psi}\right] \text{-----------}(14)$$

$$\Delta = \frac{\alpha a[(\lambda-1)(z^2-1)-1]}{R^2(\lambda-1)^2(z^2-1)^2} \text{-----------------}(15)$$

Eqs. (8) and (12) with equations. (13) - (15) comprise a set of equations relevant for determining the physical parameters exactly.

The total mass of a star of radius 'b' is given by

$$M = \frac{(a-1)b^3}{2R^2\left(1 + \frac{ab^2}{R^2}\right)} \text{--------------------}(16)$$

The Compactness factor 'u' (the ratio of mass to radius) is given by

Where $y = \frac{b}{R}$ $\quad u = \frac{M}{b} = \frac{(a-1)y^2}{2R^2(1+ay^2)} \text{--------------------}(17)$

The parameter 'B' may now be evaluated employing equations. (1), (13) and (14), which is given by

$$B = \frac{1}{4}(\rho - 3p_r) \text{--------------------}(18)$$

Where 'B' is in the unit of MeV/fm^3, obtained by scaling in

terms of a factor $\frac{3\times10^4}{R^2}$ [18], here 'R' is a constant parameter [10].

In the case of a compact star, we impose the following conditions:

1. At the boundary of the star the interior solution is matched with the Schwarzschild exterior solution, i.e.,

$$e^{2\mu}(r=b) = e^{-2\nu}(r=b) = \left(1 - \frac{2M}{b}\right) \text{-------}(19)$$

2. At the boundary of the star the radial pressure p_r should vanish, which yields,

$\frac{\Psi_z(z_b)}{\Psi(z_b)} = \frac{a-1}{2z_b} \text{------}(20)$, where $z_b = \sqrt{\frac{a}{a-1}}\left(1 + \frac{b^2}{R^2}\right)$

From eq. (12) we get $\frac{\Psi_z}{\Psi} \text{-----------}(21)$

Now using equations. (19), (20) and (21) we determine the constants C and D

3. The pressure $p_r \geq 0$ inside the star, which leads to an inequality, is given by $\frac{\Psi_z}{\Psi} \leq \frac{(a-1)}{2z}$

Physical Application

To study the effect of anisotropy of a compact star on the parameter 'B', we first use the eq. (19) to determine 'R', which can be evaluated for a given values 'a', 'M' and 'b'.

Once the constants C, D and the parameter 'R' is known, the value of 'B' can be evaluated using eq. (22) for different values of anisotropy parameter .

Thus 'B' can be studied as a function of 'r' for different values of α. 'B' is also determined from eq. (17) for a given $u = \frac{M}{b}$ and 'a'.

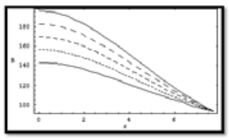
Numerical and Graphical result

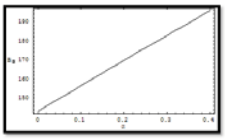
We study the interior of a compact star in two distinguished regions
(i) near the center of the star and
(ii) away from the center up to the surface.

Three different cases we have studied.

1. X Ray Pulsar HER X-1 [21]

Mass M = 0.88 $M_\odot$, where $M_\odot$ is the Solar mass
Radius b = 7.7Km. and the parameter 'a'=6, Which leads to Compactness u = 0.1686 and R= 22.8815Km.





Variations of parameter B_0 with α

Variations of parameter '**B**' with radial distant '**r**' (Km.) for different values of anisotropy Parameter α Here **u = 0.1686**, and **a = 6**. Lines from top to bottom are for α **= 0, 0.1, 0.2, 0.3, 0.4.**

In isotropic case (α = 0), B_0 = 143.05Mev/fm^3
and at the surface B_b = 93.4345Mev/fm^3.

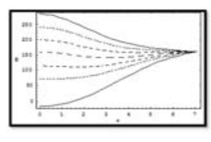
Let us consider SAX J with two possible models of compactness [16,20]

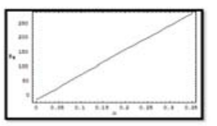
2(a). SAX J1 millisecond pulsar [16,20]

Mass M = 1.435 $M_\odot$, where $M_\odot$ is the Solar mass
Radius b = 7.07Km. and the parameter 'a'=53.34, Which leads to Compactness u = 0.2994 and R= 41.2695Km.

In Isotropic case, B_0 = -17.9387Mev/fm^3 and

At the surface B_b = 159.8817Mev/fm^3





Variations of parameter B_0 with α

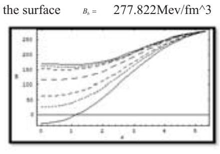
Variations of parameter 'B' with radial distant 'r' for different values of anisotropy parameter Here 'u' = 0.2979 and 'a' = 53.34. Lines from top to bottom are for α = **0, 0.1, 0.15, 0.202, 0.25, 0.3, 0.35**

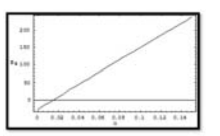
2(b). Applying scaling properties to the above configuration with scaling factor = 0.75

Mass M = 1.07625 M_0 , where $M_\odot$ is the Solar mass
Radius b = 5.3025Km. and let the parameter 'a'=20, which leads to same compactness u= 0.2994 and different value of parameter R= 18.1622Km.

In Isotropic case, B_0 = -28.1011Mev/fm^3 and

At the surface B_b = 277.822Mev/fm^3





Variations of parameter B_0 with α

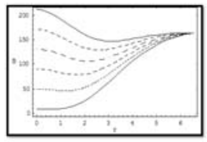
Variations of parameter 'B' with radial distant 'r' for different values of anisotropy parameter α Here 'u' = 0.2994 and 'a' = 20. Lines from top to

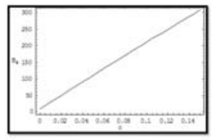
3(a). SAX J1 millisecond pulsar [16,20]

Mass M = 1.323 M_0 , where $M_\odot$ is the Solar mass
Radius b = 6.55Km. and the parameter 'a'=6, which leads to Compactness u = 0.2979 and R= 10.1288Km.

In Isotropic case, B_0 = 8.60342Mev/fm^3 and

At the surface B_b = 163.533Mev/fm^3





Variations of parameter B_0 with α

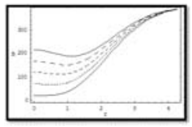
Variations of parameter 'B' with radial distant 'r' for different values of anisotropy parameter Here 'u' = 0.2979 and 'a' = 6. Lines from top to bottom are for α = **0, 0.02, 0.04, 0.06, 0.08, 0.1.**

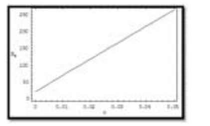
3(b). Applying scaling properties to the above configuration with scaling factor = 0.65

Mass M = 0.85995 $M_\odot$, where $M_\odot$ is the Solar mass
Radius b = 4.2575Km. and the parameter 'a'=6, which leads to Compactness u = 0.2979 and R= 6.58373Km.

In Isotropic case, B_0 = 20.3702Mev/fm^3 and

At the surface B_b = 387.06Mev/fm^3





Variations of parameter B_0 with α

Variations of parameter 'B' with radial distant 'r' for different values of anisotropy parameter Here 'u' = 0.2979 and 'a' = 6. Lines from top to bottom are for α = **0, 0.01, 0.02, 0.03, 0.04**.

M/b	**0.15**	**0.18**	**0.20**	**0.24**	**0.26**	**0.28**	**0.30**
B_o (α=0)	**2.1490**	**1.907**	**1.719**	**1.2460**	**0.934**	**0.529**	**-0.039**
B_o (α=0.3)	**2.665**	**2.566**	**2.487**	**2.289**	**2.157**	**1.987**	**1.757**
B_b	**1.406**	**1.155**	**1.0000**	**0.7200**	**0.5950**	**0.480**	**0.3750**

Variation of B_0 & B_b in unit of 3*10^4/R^2 Mev/fm^3 with compactness factor M/b for specific values of α at the center and surface of the stars having different compactness factor. Here 'R' is expressed in Km.

From the above table it is clear that at the centre of the star parameter 'B_0' decreases with an increase of 'M/b' for a given value of α , which are displayed in first two rows. Which shows that the core becomes more dense as the compactness increases, in view of the equation of state considered here.

At the surface of the star B_b also decreases with the increase of compactness factor. Which are displayed in third row.

Conclusion

* *Physically viable relativistic models of a compact star having anisotropic matter content with strange-matter EOS $p = \frac{1}{3}(\rho - 4B)$, can be obtained by following the procedure suggested by Mukherjee et al [12].*

* *In this model parameter 'B' acquired a radial dependence.*

* *At the centre of the star Parameter 'B' increases linearly with anisotropy parameter for Given compactness and spheroidicity parameter.*

* *At the surface of the star there is practically no effect of anisotropy on the value of parameter 'B'.*

* *For specific configuration of the compact object, Parameter B_0 picks up negative value, indicating the repulsive nature of core region for such configuration.*

* *Scaling properties also hold good for the model under consideration.*

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