Spin-Down Age: the Key To Magnetic Field Decay ArXiv: 1204.3445

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1 Abstract

Using the data of the ATNF pusar catalogue we study the relation connected the real pulsar age t of young NS and its spin-down age. We found that this relation is independent from both initial period of NS and its surfice magnetic field. We suppose that the laws of the surface magnetic field decay are close for all NS in the Milky Way and assume that the birth-rate of pulsars is constant during at least last 200 million years. With this assumptions we were able to restore the history of the magnetic field decay for the galactic NSs. We reconstruct the universal function f(t)=B(t)/B0, where B0 is the initial magnetic field and B(t) is the magnetic field of NS after t years. It is appeared that the function f(t) can be fitted by the power law with power index alpha = -1.17.

2 Restoration of magnetic field decay

2.1 Suppositions in base of method

1. There is an initial age when the spin-down age behavior is determined only by magnetic field behavior
2. Magnetic fields of all pulsars are changing due to uniform magnetic decay law $B = B_0 f(t)$
3. The pulsars birthrate is constant during last 10^6 years.

2.2 Algorithm of the method

1. We choose isolated non-millisecond pulsars from the ATNF catalog which lie not far than 10 kpc form the Sun.
2. Because of uniform magnetic field decay we are able to sort all the pulsars by their the spin-down ages.
3. We determine different age based on relation $t(\tau) = \frac{N(\tau)}{n_{birthrate}}$
4. The result is dependance between the spin-down age and average age
5. We fit this dependance by polynomial of high degree (6)
6. We use this polynomial in order to restore $f(t)$ by integration

2.3 Main relations

The spin-down age of a radio pulsar

$$\tau = \frac{P}{2\dot{P}}$$

Braking of the pulsar

$$P\dot{P} = \alpha B^2(t)$$

The integral expression for $f(t)$ function:

$$f(t) = \frac{\exp(\int_0^t \frac{dt'}{2\tau(t')})}{\sqrt{\tau(t)}}$$

2.4 Continuity equation

In this part we deduce formula $t(\tau) = \frac{\tau(N)}{n_{\text{birth-rate}}}$ We begin with continuity equation for ensamble of pulsars:

$$\frac{\partial n}{\partial t} + div\left(n\frac{d\tau}{dt}\right) = U - V \quad (1)$$

Integrated birthrate of pulsars:

$$R = \int_{B_{min}}^{B_{max}} \rho(B) dB \quad (2)$$

$$R = \int_{\tau_{min}}^{\tau_{max}} \rho(\tau)\frac{dB}{d\tau} d\tau \quad (3)$$

We suppose that ensemble is stationary. Therefore the continuity equation can be rewritten:

$$\frac{\partial n}{\partial \tau}\frac{d\tau}{dt} = \rho(\tau)\frac{dB}{d\tau} \quad (4)$$

We also suppose that the spin-down ages distribution function has a border at $4 \cdot 10^4$ years

$$\int_0^\tau \frac{\partial n}{\partial \tau} d\tau = \int_0^\tau \rho(\tau)\frac{dB}{d\tau}\frac{dt}{d\tau} d\tau \quad (5)$$

$$n(\tau, t) = n_{birthrate}\frac{dt}{d\tau} \quad (6)$$

$$n(\tau, t) = n_{birthrate}\frac{dt}{d\tau} \quad (7)$$

When we integrate this equation we get quantity of pulsars with the spin-down ages smaller than τ

$$N(\tau, t) = \int_0^\tau n(\tau, t) d\tau \quad (8)$$

$$t(\tau) = \frac{N(\tau)}{n_{birthrate}} \quad (9)$$

3 Stages of the Method

3.1 Observational selection

Previous investigations considered small part of Galaxy near the Sun (e.g. Lyne et al. 1998) to investigate local density of pulsars. We admit presence of severe observational selection which hide pulsars at high distances. Nevertheless, the aim of our investigation is the shape of pulsars distribution by the spin-down ages which is relative quantity. Due to this fact, for us selection is important only if it hide pulsars at certain stage of their life. Therefore, we investigate radial distribution function of pulsars with same spin-down ages. In following Figure 1 we show cumulative distribution for pulsars of different ages. It is clearly seen that selection conceals similarly pulsars with $\tau \in [1000, 2 \cdot 10^6]$ years. Older pulsars have significantly different radial distribution.

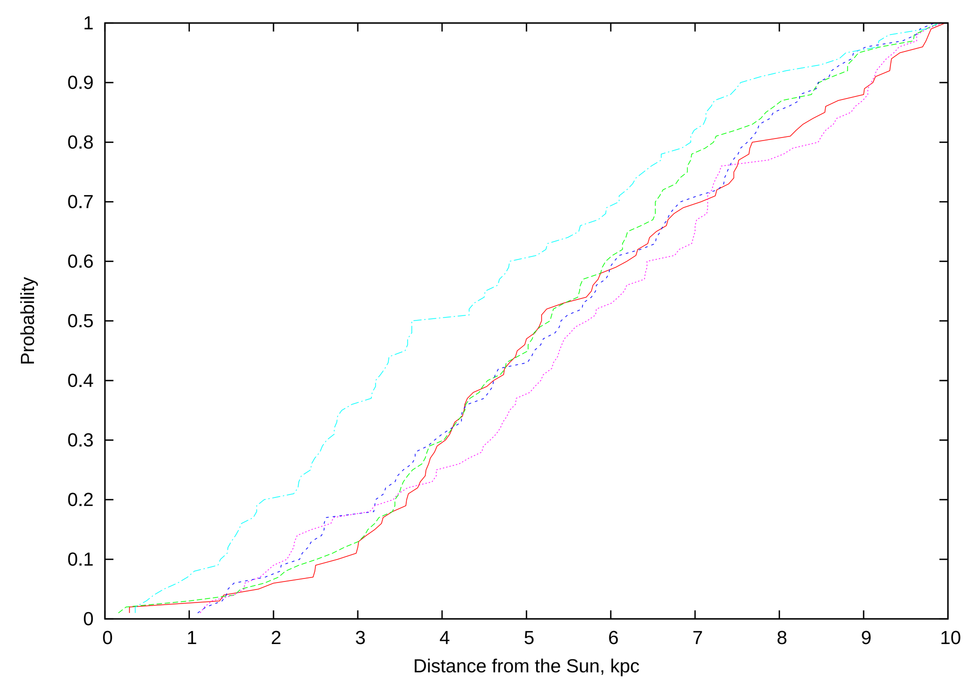

Figure 1: Cumulative distribution function of pulsars by distances from the Sun. red line - $\tau \in [700, 1.2 \cdot 10^5]$ years, green line - $\tau \in [1.2 \cdot 10^5, 4.5 \cdot 10^5]$ years, blue line - $\tau \in [4.5 \cdot 10^5, 9.6 \cdot 10^5]$ years, violet line - $\tau \in [9.6 \cdot 10^5, 1.6 \cdot 10^6]$ years and sky blue line - $\tau \in [4 \cdot 10^6, 6 \cdot 10^6]$ years. Every interval consists 100 pulsars.

3.2 Fitting of cumulative distribution

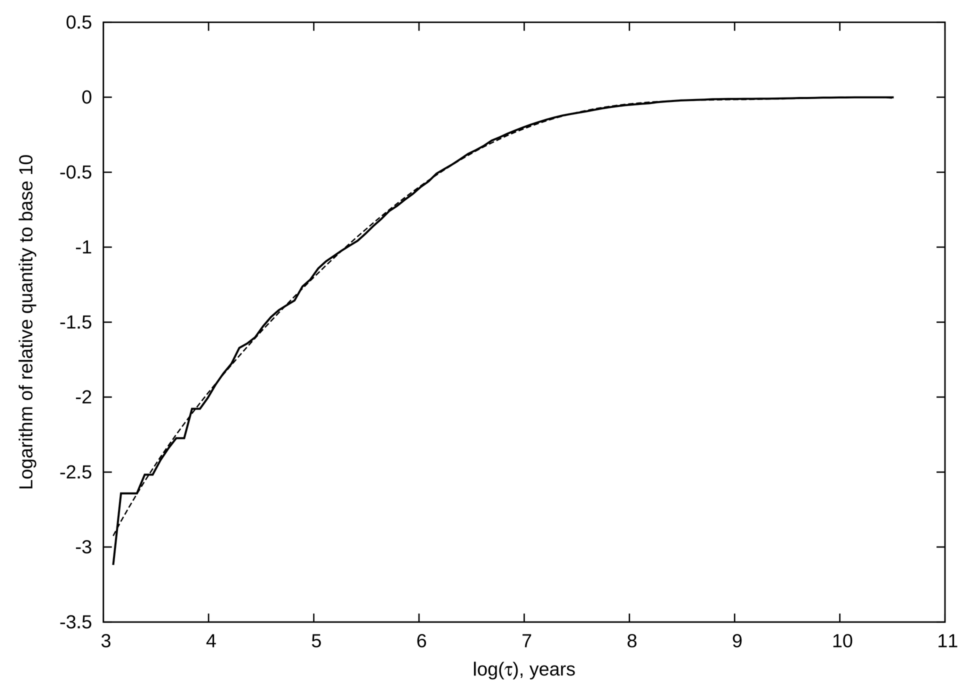

Figure 2: Cumulative distribution function of pulsars by spin-down ages fitted by polinomial

3.3 Restored curve of magnetic field decay

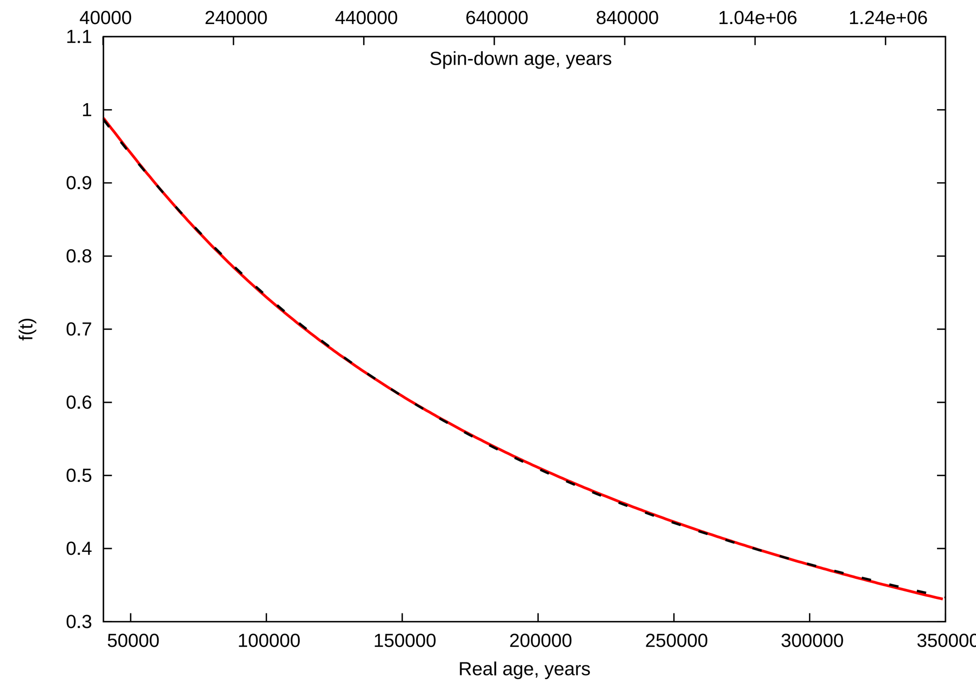

Figure 3: Restored curve of magnetic fields decay for middle aged radio pulsars.

4 Results

1. We create method for restoration of magnetic field decay law for radio pulsars
2. Method is checked on artificially generated selections
3. Magnetic field decay law is fitted by expression:

$$s(t) = \left(\left(a\frac{t}{t_0}\right)^\gamma + c\right)^{-1}$$

   Here $\gamma = 1.17$, $a = 0.034$, $c = 0.84$, $t_0 = 10000$ years.
4. We estimate the birthrate of pulsars in the Galaxy as 2.9 pulsars per century.