Particle Simulation for Axisymmetric Pulsar Magnetosphere

- Radiation drag force for accelerated plasma
- Magnetic distortion by magneto-spheric current
- Quadrupole magnetic field (Octopole induced stellar electric field)
- Magnetic pair creation & pair generation by photon-photon collision in the gap

Tomohide Wada*
Center for Computational Astrophysics, National Astronomical Observatory Japan

email: tomohide.wada @ nao.ac.jp
Particle Simulation for Axisymmetric Pulsar Magnetosphere

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Tomohide Wada*

Center for Computational Astrophysics., National Astronomical Observatory Japan

email: tomohide.wada @ nao.ac.jp
Outline of poster

**How does pulsar** (conventionally, believed magnetized rotating NS) **accelerate plasma?**

1. Observed feature of Gamma-ray Pulsar
2. Rotating magnetized neutron star
3. Aim of our study
4. Outline of particle simulation (method)
5. Results
6. Summary
We focus on Rotation Powered Pulsar

Pulsar exists inside the nebula as a central engine

Pulsar is identified highly magnetized and rapidly rotating neutron star

Gamma-ray pulsars

Image credit: NASA

Crab nebula: image credit: NASA
Gamma-ray Beam

If plasma is accelerated along the magnetic field line, it emits gamma-ray by curvature radiation.

Gamma-ray pulse; localized accelerating region corotating with star.
Axisymmetric steady model

Rotating magnetized star become unipolar dynamo!

Rotating magnetized star (conductive sphere) become unipolar dynamo. Induced E pushes out plasma from the surface. We can follow Jackson’s Gedankenexperiment (Jackson 1976 ApJ) with particle simulation.
Voltage for acceleration of plasma

- \( \phi_{\text{eff}} \): effective accelerating voltage (open field line voltage) in open zone
- \( \phi_{\text{uni}} \): voltage by unipolar induction
- \( B_0 \): surface magnetic field intensity
- \( R \): stellar radius
- \( P \): rotation period
- \( R_6 = R/(10^6 \text{ cm}) \)
- \( B_{12} = B_0/(10^{12} \text{ gauss}) \)

Formulas:

\[
\phi_{\text{uni}} = 3 \times 10^{17} B_{12} R_6 P_{0.2}^{-1} \text{ Volt}
\]

\[
\phi_{\text{eff}} = 1.6 \times 10^{14} B_{12} R_6^3 P_{0.2}^{-2} \text{ Volt}
\]
Pulsar Wind Nebula (PWN)
P-Pdot diagram

PWN; steady outflow by accelerated plasma (e+e-)
-> pair cascading in the magnetosphere
\( l_B = \frac{4.4}{\alpha \frac{\hbar}{m \gamma c B_\perp}} \exp \left( \frac{4}{3\chi} \right) \propto l_B \left( \epsilon_\gamma (\phi_{\text{eff}}, R_c), |\tilde{B}(\vec{x})| \right) \)

\( \chi = \frac{\epsilon_\gamma}{2mc^2 B_q} \)

\( B_q = \frac{m^2c^3}{c\hbar} = 4.41 \times 10^{13} \text{ G} \)

\( B_\perp = B \sin \theta \)

To maintain activity, \( l_B \ll l_{\text{mag}} \)

\[
\begin{align*}
B_{\text{dip}} & = -\frac{13}{2} \log 10P + 4 \log 10B_0 + \frac{17}{2} \log 10R \geq 96.5 \\
B_{\text{quad}} & = -\frac{14}{2} \log 10P + 4 \log 10B_0 + \frac{19}{2} \log 10R \geq 106.5
\end{align*}
\]

e.g., Kashiyama 2011 PhRvD
If multipole stellar magnetic field is considered, curvature radius along the magnetic field line become smaller than that of dipole case. As a result, the higher energy photon would be emitted and convert into pairs easily. It could maintains pair cascading with larger P star.
Motivation of our study

Aim of study is to understand the origin of pulsar wind. For this, we have to know how and where particles accelerated in the magnetosphere.

We chose particle method and focus on the effects, -radiation drag force for accelerated plasma -pair creation in the gaps -distortion of stellar field by manetospheric current -stellar multipole magnetic field
Basic equations (with *steady* condition)

To obtain steady solution, we solve Maxwell's equations in *steady state* and EOM iteratively.

\[
\begin{align*}
-\nabla^2 \phi &= 0; \text{quadrepole+monopole+octopole} \\
-\nabla^2 \phi &= 4\pi \rho \\
\text{B.C. } \phi(\vec{R}) &= \frac{\mu \Omega \sin^2 \theta}{cR} + \text{const}, \; \phi(\infty) = 0
\end{align*}
\]

\[
-\nabla^2 \vec{A} = 0; \text{dipole+quadrupole}
\]

\[
-\nabla^2 \vec{A} = \frac{4\pi}{c} \vec{j}
\]

\[
\text{B.C. } \vec{A}(\vec{R}) = \vec{e}_\varphi \frac{\mu \sin \theta}{R^2}, \quad \vec{A}(\infty) = \vec{0}
\]

**Equation of motion (3D)**

\[
m_i \frac{d}{dt} (\gamma_i \vec{v}_i) = q_i \left[ \vec{E}_i + \frac{\vec{v}_i}{c} \times \vec{B}_i \right] + \vec{f}_{\text{rad},i}
\]

Static electro-magnetic interaction is calculated by special purpose programmable computer, GRAPE-DR. CfCA web site: [http://www.cfca.nao.ac.jp](http://www.cfca.nao.ac.jp)
Treatment of pair creation

If photon moves out numerical outer boundary (10Rl), it removed.

Numerically, mean free path is estimated with \( l_p \times f \), where \( f \) is probability distribution function given by random number.

### X-gamma

\[
\begin{align*}
    l_p &= \frac{1}{n_X \sigma_p} \\
    \sigma_p &= \frac{3}{16} \sigma_T (1 - v^2) \left[ (3 - v^4) \ln \frac{1 + v}{1 - v} - 2v(2 - v^2) \right] \\
    v &= \sqrt{1 - \frac{2}{1 - \mu_c} \left( \frac{mc^2}{\epsilon} \right)^2} \\
    \mu_c &= \cos^{-1} \theta \quad \text{collision angle} \\
    \sigma_T &= \text{Thomson cross section}
\end{align*}
\]

### B-gamma

\[
\begin{align*}
    l_p &= \frac{4.4 \ \hbar}{\alpha \ mc B_\perp} \exp \left( \frac{4}{3\chi} \right) \\
    B_\perp &= B \sin \theta \quad \text{perpendicular component of magnetic field for propagating direction of} \\
    \chi &= \frac{\epsilon \gamma B_\perp}{2mc^2 B_q} \\
    B_q &= \frac{m^2c^3}{\epsilon} = 4.41 \times 10^{13} \text{ G}
\end{align*}
\]
Outline of particle method

We obtain the steady magnetosphere
- without pair creation ($B_{\text{dip}}$, $B_{\text{quad}}$)
- with pair creation

1. calculate surface charge density
2. replace surface charge density with simulation particle
3. calculate fields (stellar field plus field by space charge and current) at the position of particle and solve equation of motion for each particles
4. Gamma-ray photon is emitted in the accelerating region and convert into $e^+e^-$
5. delete particle outside of the outer boundary
6. repeat 1-4 until steady state is obtained

ongoing work

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Simulations

We carried out three cases of magnetic field, that is pure dipole case, equivalent case in which intensity of dipole field and quadrupole field has same at the pole and quadrupole dominant case. After obtaining steady state with no pair case, pair creation model is carried out successively. For pair creation, energy dependence of photon is considered, but the multiplicity is fixed low value artificially. “X-gamma”: photon-photon collision process “B-gamma”: magnetic pair creation

<table>
<thead>
<tr>
<th></th>
<th>no pair</th>
<th>X-gamma</th>
<th>B-gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{dip}$</td>
<td>done</td>
<td>ongoing</td>
<td>ongoing</td>
</tr>
<tr>
<td>$B_{quad}$</td>
<td>done</td>
<td>not-yet</td>
<td>not-yet</td>
</tr>
<tr>
<td>$B_{dip}=B_{quad}$</td>
<td>done</td>
<td>not-yet</td>
<td>not-yet</td>
</tr>
<tr>
<td>$B_{dip}&lt;B_{quad}$</td>
<td>done</td>
<td>not-yet</td>
<td>ongoing</td>
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</tbody>
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Result 1: pair suppressed case

Static electrosphere (polar domes and equatorial disc) + weak outflow of plasma (diocotron instability and $f_{\text{rad}} \times B$ drift)
Remark: $B_{\text{quad}}$ dominant case

If pair plasma is suppressed and $B_{\text{quad}}$ is dominant near the star rather than $B_{\text{dip}}$,

- asymmetric stellar field ($B_p$, induced $E_p$)
- (quasi-)static electrosphere, no wind
- asymmetric accelerating region on both sides of equatorial plane (see, right panel)
- the number of returning plasma might differ in the north and south hemisphere
Result 2: test run with pairs

Initially, pair cascades in outer gaps, negative charges return star and are emitted from pole again. And then, the potential drop is made polar region. We only carried out this model in one rotation period. To obtain final state is challenges for future,
Summary

We demonstrated the particle simulation for axisymmetric pulsar magnetosphere. As a result of test run,
1. If pair creation is suppressed, quasi static state is obtained as previous work (Wada & Shibata 2011).
2. Multipole B field makes asymmetric electrosphere to the equatorial plane (i.e. symmetric outer gaps)
3. Negative charge returns star and potential arise from polar region

Our simulation has just started. In the future work,
• Detailed structure of gaps
  (Can conventional gap models are coexist?; Yuki & Shibata 2012 PASJ)
• Investigation for higher multiplicity and Lower $E_{cr}$ model
• Simultaneous effect of B-gamma and X-gamma
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-CfCA: http://www.cfca.jp
-4D2U: http://www.4d2u.nao.ac.jp/
End

Thanks